Negative Refractive Index Metamaterials Supporting 2-D Waves

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Abstract – **Recent demonstrations of negative refraction utilize three-dimensional collections of discrete periodic scatterers to synthesize artificial dielectrics with simultaneously negative permittivity and permeability. In this paper, it is shown that planar, two-dimensional L-C transmission line networks in a high pass configuration can demonstrate negative refraction as a consequence of the fact that such media support propagating backward waves. Simulations illustrating negative refraction and focusing at 2 GHz are subsequently presented.**

I. Introduction

The concept of a negative index of refraction, originally proposed by Veselago in the 1960s [1], suggested the possibility of materials in which the permittivity and permeability could be made simultaneously negative. Veselago termed these Left-Handed Media (LHM), because the vectors **E**, **H**, and **k** would form a left-handed triplet instead of a right-handed triplet, as is the case in conventional, Right-Handed Media (RHM). Recently, novel 3-D electromagnetic materials have successfully demonstrated negative refraction by synthesizing a negative refractive index. These artificial dielectrics (metamaterials) consist of loosely coupled unit cells comprised of thin wire strips and split ring resonators to synthesize negative permittivity and permeability, respectively [2]-[4]. In these metamaterials, the choice of operating frequency is restricted to the region above the ring resonance, which requires relatively large unit cells.

In this paper, it is proposed that planar transmission line networks supporting propagating backward waves exhibit left-handedness, and can therefore be used to demonstrate negative refraction and focusing. Such planar transmission line networks inherently support 2-D wave propagation, which is desirable for microwave circuit applications. They may be equipped with lumped elements, which permits them to be compact even in the lower RF range. Furthermore, these networks do not rely on resonances, and their left-handedness stems from their ability to support backward waves. To illustrate this concept, consider a phase-matching argument at the interface between a right-handed medium M1 and another generic medium M2, as shown in Figure 1.

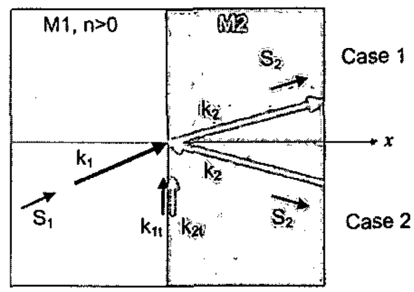

Fig. 1: Phase Matching at the Interface Between a Right-Handed Medium M1 and a Generic Medium M2

We are unconcerned for the moment as to the sign of the index of refraction of M2. Consider an incident plane wave in M1 with a wavevector $\mathbf{k}_1$ (i.e., such that the x-component of $\mathbf{k}_1$ is positive); then a refracted wave with a wavevector $\mathbf{k}_2$ is established in M2 such that the tangential wavevector components $\mathbf{k}_{1t}$ and $\mathbf{k}_{2t}$ are equal across the interface. This is the basis for Snell's Law, and it permits two scenarios for the orientation of $\mathbf{k}_2$, represented as Case 1 and Case 2 in Figure 1. The conservation of energy also insists that the normal components of the Poynting vectors $\mathbf{S}_1$ and $\mathbf{S}_2$ remain in the positive-x direction through both media. If the generic medium M2 is a conventional RHM, then refraction occurs as illustrated by Case 1 in Figure 1. However, if M2 is a medium supporting propagating backward waves (LHM), it is implied that power is propagated along the direction of phase advance, which requires in Figure 1 that $\mathbf{k}_2$ and $\mathbf{S}_2$ be antiparallel. Consequently, the direction of $\mathbf{k}_2$ is specified uniquely for backward-wave structures as illustrated by Case 2 in Figure 1. Under such conditions, power is refracted through an effectively negative angle, which indeed implies an effectively negative index of refraction.

II. Proposed Structure

It has long been known that transmission lines periodically loaded with capacitive (C) and inductive (L) elements in a high-pass configuration support one-dimensional backward waves [5],[6]. In particular, the *fundamental spatial harmonic accumulates positive phase* as it carries power from left (source) to right. Consequently, the analysis exposed in the introduction prompts the examination of 2-D loaded transmission line networks as a means for synthesizing an equivalent negative refractive index. A node representing the unit cell of such a 2-D medium is shown in Figure 2, where Z_C and Y_L represent the per-unit-length capacitor impedance and inductor admittance, respectively.

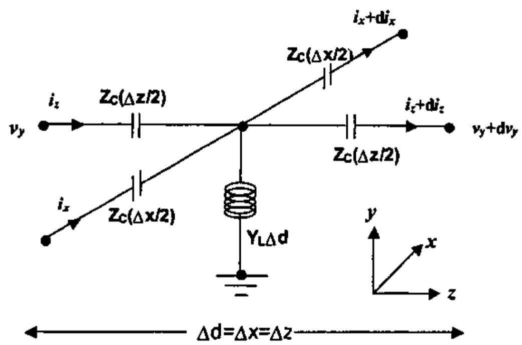

Fig. 2: 2-D Loaded High-Pass Transmission Line Unit Cell

The cell dimensionality represented by Δd is provided by the host transmission line medium. At the continuous medium limit, Δd<<λ, the 2-D telegrapher's equations representing the structure of Figure 2 can be expressed as

$$\frac{\partial v_y}{\partial z}=-i_z\left(\frac{1}{j\omega C\Delta d}\right),\ \frac{\partial v_y}{\partial x}=-i_x\left(\frac{1}{j\omega C\Delta d}\right) \quad (1)$$

and

$$\frac{\partial i_z}{\partial z}+\frac{\partial i_x}{\partial x}=-v_y\left(\frac{1}{j\omega L\Delta d}\right). \quad (2)$$

Combining (1) and (2) yields

$$\frac{\partial^2 v_y}{\partial x^2}+\frac{\partial^2 v_y}{\partial z^2}+\beta^2 v_y=0,\ \beta=-\frac{1}{\omega\sqrt{LC}\Delta d} \quad (3)$$

where β is the propagation constant. The phase and group velocities are antiparallel and are given by

$$v_\phi=\frac{\omega}{\beta}=-\omega^2\sqrt{LC}\Delta d,\ v_g=\left(\frac{\partial\beta}{\partial\omega}\right)^{-1}=+\omega^2\sqrt{LC}\Delta d\cdot \quad (4)$$

Subsequently, a *negative* refractive index can be defined as

$$n=\frac{c}{v_\phi}=-\frac{1}{\omega^2\sqrt{LC}\sqrt{\mu_0\varepsilon_0}\Delta d}\cdot \quad (5)$$

It is interesting to note that it is possible to achieve the same result for the phase velocity and index of refraction if an equivalent negative permittivity and permeability are defined as

$$\mu_{eq}=-\frac{1}{\omega^2 C\Delta d},\ \varepsilon_{eq}=-\frac{1}{\omega^2 L\Delta d}, \quad (6)$$

so that the total stored time-averaged energy [7], expressed by

$$W=\frac{\partial(\mu_{eq}\omega)}{\partial\omega}|\mathbf{E}|^2+\frac{\partial(\varepsilon_{eq}\omega)}{\partial\omega}|\mathbf{H}|^2, \quad (7)$$

remains positive.

III. DESIGN

In order to model the 2-D wave equation represented by (3), an array of LHM unit cells, each as depicted in Figure 2, was implemented in the HP-ADS circuit simulator. To simulate the incidence of waves on this LHM in a circuit environment, it is necessary to design a RHM as well. The topology of the RHM unit cell is identical to that of Figure 2, except that the roles of C and L are interchanged, giving rise to the well-known L-C low-pass network representation of free-space propagation.

The specification of the unit cell parameters in both media provides information about the permissible operating frequencies, the relative refractive indices, and also the required inductance and capacitance values. In both media, the wave impedance is given by

$$Z_M=\sqrt{\frac{L}{C}}\cdot \quad (8)$$

It is therefore reasonable to begin with the simple constraint that the two media be matched, and moreover, matched to free space (Z_M = 377 Ω).

In the limit Δd<<λ in (5), it is not possible to directly specify indices of refraction in the individual media, but it is possible to specify a relative index of refraction through the ratio of their respective phase shifts per unit cell $\beta_{LHM}\Delta d/\beta_{RHM}\Delta d$. The LHM is designed to be denser than the RHM, arbitrarily by a factor of 2; specifically, phase shifts per unit cell of $|\beta_{RHM}\Delta d|$=0.25 and $|\beta_{LHM}\Delta d|$=0.5 are chosen in the right-handed and left-handed media, respectively. Choosing an operating frequency of 2 GHz, the LHM and RHM unit cell capacitors and inductors are specified as (C_{LHM}=422.5fF, L_{LHM}=60.0nH) and (C_{RHM}=52.8fF, L_{RHM}=7.5nH), respectively. The corresponding cut-off frequencies are given by

$$f_{c,LHM}=\frac{1}{4\pi\sqrt{L_{LHM}C_{LHM}}},\ f_{c,RHM}=\frac{1}{\pi\sqrt{L_{RHM}C_{RHM}}}, \quad (8)$$

which are found to be 500 MHz and 16 GHz, respectively. Finally, the designed LHM and RHM arrays are appropriately terminated with matching resistors on all edges and simulated with the HP-ADS microwave circuit simulator.

IV. RESULTS

A. Negative Refraction

To verify the ray picture presented in the phase-matching argument of Figure 1, a RHM/LHM interface was constructed using 42×42-cell RHM and LHM arrays with $\beta_{RHM}\Delta d$ = +0.25 and $\beta_{LHM}\Delta d$ = –0.5, yielding a relative refractive index of –2. The RHM array was excited with a plane wave, which was simulated using sequentially phase-shifted voltage sources along the left boundary of the array. One such excitation is shown in Figure 3, for an incident angle of θ_{RHM}=29°. The steepest phase descent in the LHM is observed along the direction of propagation, which is θ_{LHM}=–14° from the normal, in exact correspondence with Snell's Law for the given design parameters.

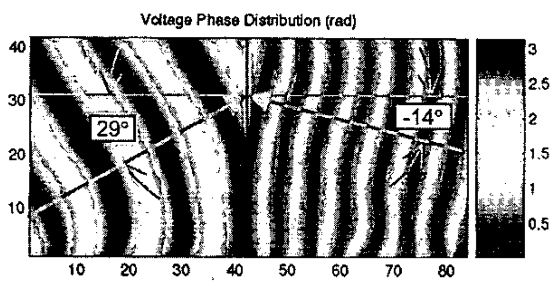

Fig. 3: Plane wave illuminating a RHM/LHM interface at 2 GHz (relative refractive index –2, homogeneous limit with Δd<<λ), negative refraction observed; the axes are labeled according to cell number, and the right vertical scale designates radians.

B. Focusing

According to [1],[8] electromagnetic waves from a point source located inside a RHM can be focused inside a LHM using a planar interface of the two media, as depicted in Figure 4. These conditions can be modeled by exciting a single node inside the RHM and observing the magnitude and phase of the voltages to ground at all points in the LHM. A focusing effect should manifest itself as a "spot" distribution of voltage at a predictable location in the LHM.

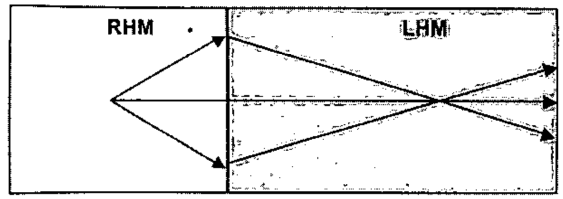

Fig. 4: Illustration of the Focusing Effect at an Interface between Right- and Left-Handed Media

Hereinafter, *a more practical* version of the medium used to generate the results of Figure 3 is examined, which is constructed by inserting *finite length* transmission line sections (k, Z_0, finite Δd) in each L-C unit cell. In order that this host transmission-line medium does not significantly alter the propagation constant predicted by (5), it is necessary to modify the loading elements C_{LHM} and L_{LHM} to compensate for the presence of the distributed transmission line parameters. In the final design, the lines in each unit cell are designed to be air-filled, with Δd=5mm. From (5), and using the appropriately compensated loading element values, the corresponding equivalent, absolute index of refraction of the LHM is approximately equal to –2.4. To maintain the relative refractive index of –2, the absolute index of refraction of the RHM is made to be +1.2.

In the interest of completeness, cases of both positive and negative refraction are examined, with the host transmission line medium in place. In the first case, a 42×21-cell RHM array is interfaced with another 42×63-cell RHM array with a relative refractive index of +2. The source is placed 11 cells into the first RHM. Here, focusing is not expected since Snell's Law for positive-index media predicts a continued divergence into the second RHM. Figure 5 presents magnitude and phase plots of these results, and confirms that the cylindrical wave excitation diverges into the second medium.

The second case is the RHM/LHM interface with a relative refractive index of –2, this time using the array dimensions and source location specified above. This arrangement is expected to show focusing inside the LHM, in accordance with Figure 4. The paraxial limit dictates a focus in the LHM at twice the distance of the source from the interface, or near cell 44 of the composite array. Indeed, as shown in Figure 6, the corresponding magnitude and phase results show focusing in this region, manifested in increased voltage amplitudes (nearly 65% of the source amplitude), and also in the reversal of the concavity of the wave fronts at both the RHM/LHM boundary and the expected focal point.

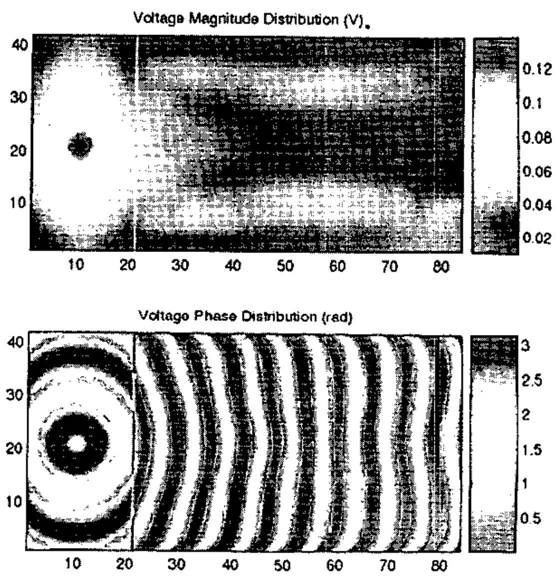

Fig. 5: Point source illuminating a RHM1/RHM2 interface at 2 GHz (relative refractive index +2, finite air-filled transmission lines with Δd=5mm included in each unit cell), no focusing observed in either phase or magnitude; the axes are labeled according to cell number.

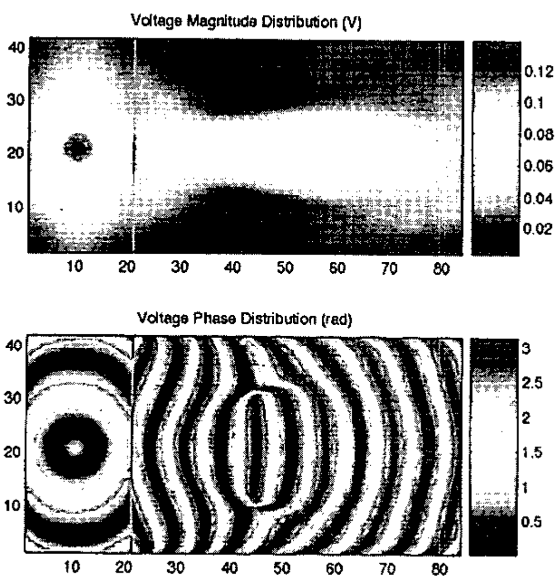

Fig. 6: Point source illuminating a RHM/LHM interface at 2 GHz (relative refractive index –2, finite air-filled transmission lines with Δd =5mm included in each unit cell), focusing observed in both phase and magnitude; the axes are labeled according to cell number.

V. Conclusion

The ability to refract radiation at negative angles can have numerous implications, essentially providing the means to reinvent electromagnetic and opto-electronic devices and equipment. In this paper, it has been shown that negative refraction and focusing of electromagnetic waves can be achieved in periodically loaded 2-D transmission line networks that support backward waves without employing resonances or directly synthesizing the permittivity and permeability. We have also suggested a practical scheme for fabricating such media by appropriately loading a host transmission line medium. The resulting *planar* topology permits LHM structures to be readily integrated with conventional planar microwave circuits and devices.

Acknowledgement

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